AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.	1 : answer
(c) $h(0) = 0$ For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$ For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$ For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$ Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$	$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$
(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.	$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

1A1

A graphing calculator is required for some problems or parts of problems.

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Work for problem 1(a)

$$\int_0^6 f(t)\,dt$$

$$\int_0^6 7te^{\cos t}\,dt$$

$$= 142.275 \text{ ft}^3$$

Work for problem 1(b)

$f(t) - g(t)$ at 8 am

$7te^{\cos t} - 108$ cubic feet per hour

$7(8)e^{\cos 8} - 108$

$= -59.583 \text{ ft}^3/\text{hr}$

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Continue problem 1 on page 5.

Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 108(t-7)+125 & \text{for } 7 < t \le 9 \end{cases}$$

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Work for problem 1(d)

$$\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$$

$$\int_0^9 7te^{\cos t}\,dt - h(t)\Big|_0^9$$

$$367.334 - (125 + 216)$$

$= 26.334$ ft^3 of snow are on the driveway at 9 am.

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CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

1B,

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Rate of accumalation of snow $= 7te^{\cos t}$

Accumulation at 6 A.M $= \int_0^6 (7te^{\cos t})\,dt$

≈ 142.275 ft^3

Work for problem 1(b)

Volume of snow at 8 A.M $= 7te^{\cos t} - \int 108$

$\frac{dV}{dt} = (7t)(e^{\cos t} \cdot -\sin t) + (7)(e^{\cos t})$

$\frac{dV}{dt} = 7t(-e^{\cos t}\sin t) + 7e^{\cos t}$

$\frac{dV}{dt} = -7te^{\cos t}\sin t + 7e^{\cos t}$

At $t=8$, $\frac{dV}{dt} \approx -41.8496$ ft^3/hr

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Continue problem 1 on page 5.

Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125t & \text{for } 6 \le t < 7 \\ 108t & \text{for } 7 \le t \le 9 \end{cases}$$

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Work for problem 1(d)

Total amount of snow falling from $0 \le t \le 9$

$$= \int_0^9 (7te^{\cos t})\,dt \approx 367.33461 \text{ ft}^3$$

From $6 \le t < 7$, Janet removed:

$$\int_6^7 125\,dt = 125 \text{ ft}^3$$

From $7 \le t \le 9$, Janet removed

$$\int_7^9 108\,dt = 216 \text{ ft}^3$$

So, at $t = 9$, total snow $= (367.33461) - (125) - (216) \approx 26.335 \text{ ft}^3$

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

1C1

A graphing calculator is required for some problems or parts of problems.

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Work for problem 1(a)

$f(t) = 7te^{\cos t}$

$\int_0^6 (7te^{\cos t})\,dt = 142.275 \text{ ft}^3$

Work for problem 1(b)

$f(8) = 7(8)e^{\cos 8}$

$= 48.417 \text{ ft}^2/h$

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Continue problem 1 on page 5.

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Work for problem 1(c)

$$\int_6^7 125\,dt = 125$$

$$\int_7^9 108\,dt = 216$$

$$h(t)\begin{cases} 0, & 0 \le t < 6 \\ 125, & 6 \le t < 7 \\ 216, & 7 \le t \le 9 \end{cases}$$

Work for problem 1(d)

$$\int_0^9 (7te^{\cos t})\,dt = 367.335\ ft^2$$

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AP® CALCULUS AB
2010 SCORING COMMENTARY

Question 1

Overview

This problem supplied two rate functions related to the amount of snow on Janet's driveway during a nine-hour period. One function f, given by $f(t) = 7te^{\cos t}$, measured in cubic feet per hour, models the rate of accumulation on the driveway for t between 0 and 9 hours after midnight. A second function, g, is a step function that gives the rate at which Janet removes snow from the driveway during this period. For part (a) students needed to use the definite integral $\int_0^6 f(t)\,dt$ to calculate the accumulation of snow on the driveway by 6 A.M. — integrating the rate of accumulation of snow over a time interval gives the net accumulation of snow during that time period. Part (b) asked for the rate of change of the volume of snow on the driveway at 8 A.M.; students needed to recognize this as the difference $f(8) - g(8)$ between the rate of accumulation and the rate of removal at time $t = 8.$ Part (c) asked the students to recover a function h measuring the total amount of snow removed from the driveway for t between 0 and 9 hours after midnight. Students needed to integrate to obtain a piecewise-linear expression for h from the step function g. Part (d) asked for the amount of snow on the driveway at 9 A.M., which required students to compute the difference of two integrals, $\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt.$

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student's work is correct. In part (b) the student works with f', rather than f and g. The student's numeric answer is incorrect. In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The second point was not earned since the student reports that the linear expression is $125t.$ The student does not use the initial condition that $h(7) = 125$ and does not horizontally translate the linear expression, so the third point was not earned. In part (d) the student's work is correct.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not subtract $g(8)$ from the evaluation of $f(8).$ In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The student presents constant functions for the other intervals and did not earn the other two points. In part (d) the student earned the point for the correct integral expression.

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a) $E'(6) \approx \dfrac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour

1 : answer

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$

$$\frac{1}{8}\left(2\cdot\frac{E(0)+E(2)}{2} + 3\cdot\frac{E(2)+E(5)}{2} + 2\cdot\frac{E(5)+E(7)}{2} + 1\cdot\frac{E(7)+E(8)}{2}\right)$$

$= 10.687$ or 10.688

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.

$3 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{meaning} \end{cases}$

(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

$3 : \begin{cases} 1 : \text{considers } P'(t) = 0 \\ 1 : \text{identifies candidates} \\ 1 : \text{answer with justification} \end{cases}$

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

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Work for problem 2(a)

$$\frac{dE(6)}{dt} \approx \frac{E(7)-E(5)}{7-5}$$

$$= \frac{21-13}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

about 400 entries per hour at $t=6$.

Work for problem 2(b)

$$\frac{1}{8}\int_0^8 E(t)\cdot dt \approx \frac{1}{8}\left[\frac{1}{2}(2-0)(4+0)+\frac{1}{2}(5-2)(4+13)+\frac{1}{2}(7-5)(21+13)+\frac{1}{2}(8-7)(23+21)\right]$$

$$= \frac{1}{8}\left[\cancel{\tfrac{1}{2}}\cancel{(2)}(4)+\frac{1}{2}(3)(17)+\cancel{\tfrac{1}{2}}\cancel{(2)}(34)+\frac{1}{2}(1)(44)\right]$$

$$= \frac{1}{8}\left(4+\frac{51}{2}+34+22\right) = \frac{1}{8}\left(\frac{171}{2}\right) = \frac{171}{16} \text{ (or } 10.6875)$$

$\frac{1}{8}\int_0^8 E(t)\cdot dt$ is approximately 10.6875 [hundred entries].

$\frac{1}{8}\int_0^8 E(t)\cdot dt$ signifies the average value of the number of entries over the interval $0 \le t \le 8$ in hundreds

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Continue problem 2 on page 7.

Work for problem 2(c)

# of processed entries = $\int_8^{12} p(t)\,dt$

$= \int_8^{12} (t^3 - 30t^2 + 298t - 976)\,dt = 16$

∴ the # of unprocessed entries is $(E(8) - 16)$* which is $23 - 16 = 7$
*in hundreds

$\boxed{\text{7 hundred entries had not yet been processed}}$

Work for problem 2(d)

when P(t) is at maximum, ~~the~~ rate of process is the ~~fastest~~

→ p(t) @ local max, when $P'(t) = 0$ and $P''(t) < 0$

$p'(t) = 3t^2 - 60t + 29\cancel{5} = 0 \rightarrow t = 9.1835$ or $t = 10.8165$

$p''(t) = 6t - 60$

$p''(9.1835) = 6(9.1835) - 60 < 0$

$p''(10.8165) = 6(10.8165) - 60 > 0$ ∴ $t = 10.8165$ is not valid

→ p(t) at end points (aka $t = 8$ or $t = 12$)

$p(8) = 0$

$p(12) = 8$ $p(12) > p(9.1835)$

and $p(9.1835) = 5.08866$

∴ p(t) is at maximum at $t = 12$.

$\boxed{\text{at midnight, the entries are being processed most quickly}}$

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

Work for problem 2(a)

$$\frac{E(7) - E(5)}{7-5} = \frac{21-13}{2} = 4 \text{ hundreds/hr}$$

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Work for problem 2(b)

$$\frac{1}{8}\int_0^8 E(t)\,dt$$

$$= \frac{1}{8}\left[\frac{(0+4)(2-0)}{2} + \frac{(4+13)(5-2)}{2} + \frac{(13+21)(7-5)}{2} + \frac{(21+23)(8-7)}{2}\right]$$

≈ 10.688 hundreds of entries

The average numbers of entries is about 10.688 hundreds

Continue problem 2 on page 7.

Work for problem 2(c)

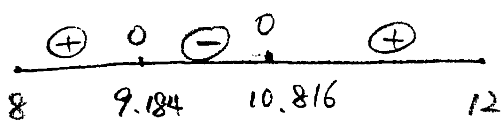
$\int_8^{12} P(t)\,dt = 16$ hundreds

$23 - 16 = 7$ hundreds

Work for problem 2(d)

$P'(t) = 3t^2 - 60t + 298$

$P'(t) = 0 \Rightarrow t \approx 9.184 \quad t \approx 10.816$



At $t = 9.184$, the entries were being processed most quickly because $P'(t) = 0$ at $t = 9.184$ and $P'(t)$ changes from positive to negative, which means local maximum occurs at $t = 9.184$, which was the time that entries processed most quickly

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

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Work for problem 2(a)

$$\frac{E(7)-E(5)}{7-5} = \frac{21-13}{7-5} = 4$$

$\boxed{\approx 400 \text{ entries per hour}}$

Work for problem 2(b)

$$\frac{(2-0)\left(\frac{4-0}{2}\right)+(5-2)\left(\frac{13-4}{2}\right)+(7-5)\left(\frac{21-13}{2}\right)+(8-7)\left(\frac{23-21}{2}\right)}{8}$$

$\boxed{\approx 331.25 \text{ entries per hour}}$

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average rate of entries per hour.

Continue problem 2 on page 7.

Work for problem 2(c)

$23 - \int_8^{12} P(t)\,dt = \boxed{700 \text{ entries}}$

Work for problem 2(d)

$t=8$ $P(8)=0$
$t=9.1835$ $P(9.1835)=5.087$
$t=12$ $P(12)=8$

At $t=12$, the entries were being processed the quickest because the rate of change is largest there

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Question 2

Overview

This problem involved a zoo's contest to name a baby elephant. Students were presented with a table of values indicating the number of entries $E(t)$, measured in hundreds, received in a special box and recorded at various times t during an eight-hour period. Part (a) asked for an estimate of the rate of deposit of entries into the box at time $t = 6$. Students needed to recognize this rate to be the derivative value $E'(6)$. Since $t = 6$ falls between the time values specified in the table, students needed to calculate the average rate of change of E across the smallest time subinterval from the table that brackets $t = 6$. Part (b) asked for an approximation to $\frac{1}{8}\int_0^8 E(t)\,dt$ using a trapezoidal sum and the subintervals of $[0, 8]$ indicated by the data in the table. Students were further asked to interpret this expression in context, with the expectation that they would recognize that it gives the average number of hundreds of entries in the box during the eight-hour period. In part (c) a function P was supplied that models the rate at which entries from the box were processed, by the hundred, during a four-hour period ($8 \le t \le 12$) that began after all entries had been received. This part asked for the number of entries that remained to be processed after the four hours. Students needed to recognize that the number of entries processed is given by $\int_8^{12} P(t)\,dt$, so that the number remaining to be processed, in hundreds of entries, is given by the difference between the total number of entries in the box, $E(8)$, as given by the table, and the value of this integral. Part (d) cited the model $P(t)$ introduced in the previous part and asked for the time at which the entries were being processed most quickly. Students should have recognized this as asking for the time corresponding to the maximum value of $P(t)$ on the interval $8 \le t \le 12$ and applied a standard process for optimization on a closed interval.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student sets up a correct difference quotient based on the values in the table and correctly evaluates for the numerical answer. In part (b) the student sets up a correct trapezoidal sum and evaluates it based on the data in the table to obtain a correct approximation. The student did not earn the third point in part (b) because the meaning given does not address the time interval over which the average was computed. In part (c) the student earned both points. The first point was earned for correctly providing the definite integral that represents the number of hundreds of entries processed between 8 P.M. and midnight. The second point was earned for subtracting that value from the initial condition of 23 hundred entries in the box at 8 P.M. to obtain the answer. In part (d) the student earned the first point for setting $P'(t) = 0$. The student does not consider the endpoints as candidates, so no additional points were earned.

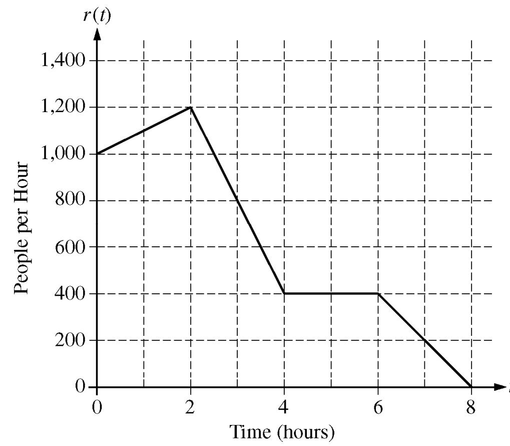
Question 2 (continued)

Sample: 2C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student sets up a correct difference quotient based on the values in the table and correctly evaluates for the numerical answer. In part (b) the student subtracts the function values at endpoints of the subintervals rather than adding them. The student interprets the integral expression as an average rate rather than an average number. In part (c) the student's work is correct. "700" was accepted because of the units in this question. In part (d) the student never considers $P'(t)$, so no points were earned.

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 3



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

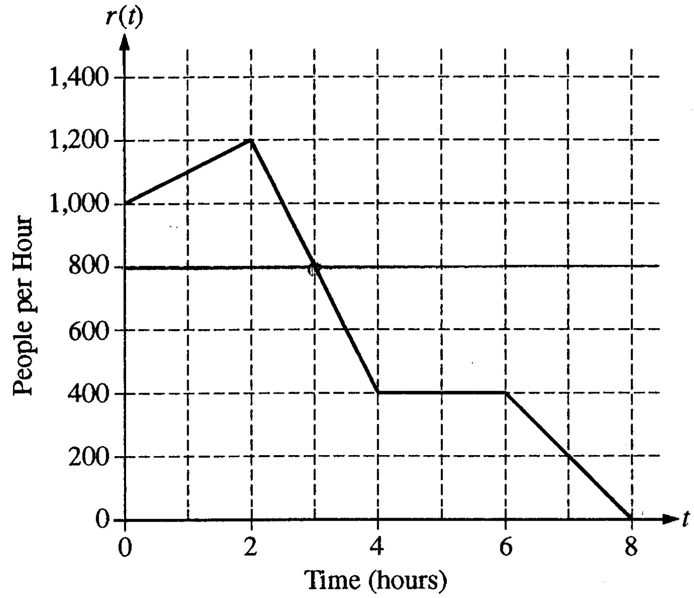
(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2} + \frac{1200+800}{2} = 3200$ people	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2 < t < 3$, $r(t) > 800$.	1 : answer with reason
(c) $r(t) = 800$ only at $t = 3$ For $0 \le t < 3$, $r(t) > 800$. For $3 < t \le 8$, $r(t) < 800$. Therefore, the line is longest at time $t = 3$. There are $700 + 3200 - 800\cdot 3 = 1500$ people waiting in line at time $t = 3$.	$3: \begin{cases} 1: \text{identifies } t = 3 \\ 1: \text{number of people in line} \\ 1: \text{justification} \end{cases}$
(d) $0 = 700 + \int_0^t r(s)\,ds - 800t$	$3: \begin{cases} 1: 800t \\ 1: \text{integral} \\ 1: \text{answer} \end{cases}$



Work for problem 3(a)

Initially 700 ppl present

$\int_0^3 r(t)\,dt$ ← amount arrived from t=0 to t=3

$= \frac{(1000+1200)\times 2}{2} + \frac{(800+1200)\times 1}{2}$

= 3200 ppl

Work for problem 3(b)

Between t=2 and t=3
The rate of ppl arriving is greater than the rate in which ppl move onto the ride
Therefore the number of ppl waiting in line is increasing between t=2 and t=3.

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Continue problem 3 on page 9.

Work for problem 3(c)

The line is the longest at $t=3$ since from $t=0$ to $t=3$ $r(t) >$ the rate ppl move onto the ride. From $t=3$ to $t=8$, $r(t) <$ the rate ppl move onto the ride so the lineup will be shorter

Amount of ppl in line

$700 + \int_0^3 r(t)\,dt - (800 \times 3)$

$= 3900 - 2400$

$= 1500$ ppl in line at $t=3$

Work for problem 3(d)

$700 + \int_0^t r(x)\,dx - \int_0^t 800\,dx = 0$

or

$700 + \int_0^t (r(x) - 800)\,dx = 0$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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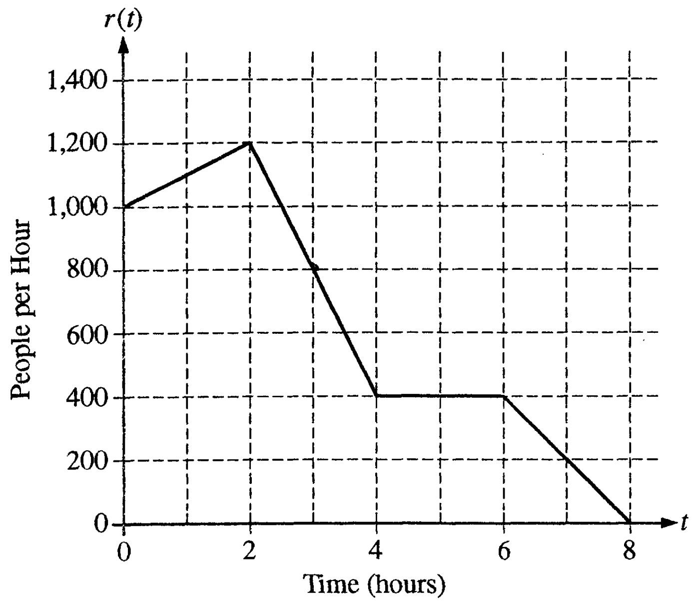

Work for problem 3(a)

$\int_0^3 r(t)\,dt = 3{,}200$

Work for problem 3(b)

increasing because the people boarding the ride remains constant while the people arriving at the line is still greater than the 800 people per hour boarding

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Continue problem 3 on page 9.

3B2

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Work for problem 3(c)

total people in line = $700 + \int_0^t r(t)\,dt - 800t$

at time $t=3$ because the rate of people boarding there the overtakes the number arriving at the line

$800+700$

1,500 people are in line at time $t=3$

Work for problem 3(d)

$700 + \int_0^t r(t)\,dt - 800t$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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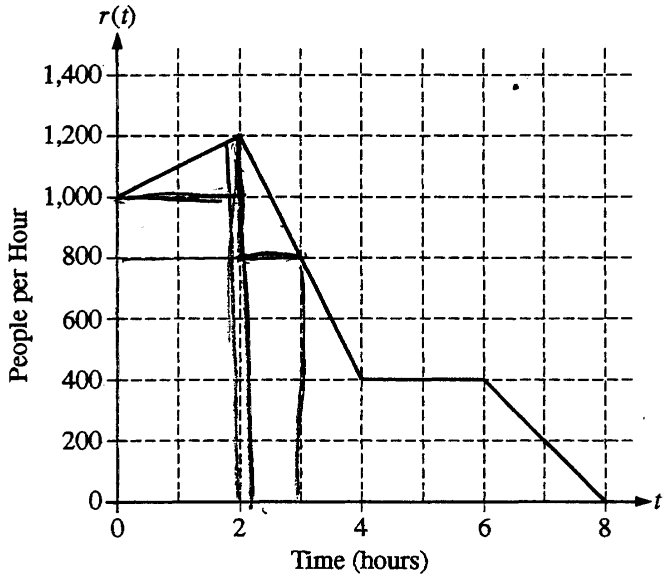

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Work for problem 3(a)

$\int_0^3 r(t)\,dt$

$= \frac{2(200)}{2} + \frac{1(400)}{2} + 2(200) + 3(800)$

$=$ 3200 people arrive at the ride between $t=0$ and $t=3$

Work for problem 3(b)

$-\int_2^3 800\, d(t) + \int_2^3 r(t)\,dt$

$-800 + \frac{1(400)}{2} + 1(800)$

$= 200$

The number of people waiting in line to get on the ride is increasing because the rate at which people is decreasing.

Continue problem 3 on page

3C2

Work for problem 3(c)

$\frac{dy}{dt}\left(\int_0^t r(t)dt - \int_0^t 800\,dt\right)$

$= \int_0^2 r(t)dt - \int_0^2 800\,(dt)$

$= 2{,}200 - 1600$

$= 600$

There are 600 people in line at time $t=2$.

At time 2, the line for the ride is the longest because $r(t)$ changes from + to −

Work for problem 3(d)

$0 = \int_0^t r(t)dt - \int_0^t 800\,dt$

Do not write beyond this border.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Overview

The context for this problem was the line for an amusement-park ride. It was given that 700 people were in line when the ride began operation in the morning, and the rate $r(t)$, in people per hour, at which people join the ride was supplied via a piecewise-linear graph for $0 \le t \le 8.$ It was also given that during the eight hours the ride is in operation, people move onto it at the rate of 800 people per hour, provided there are people waiting. Part (a) asked for the number of people arriving at the ride between times $t = 0$ and $t = 3$ hours. Students needed to obtain this value by computing $\int_0^3 r(t)\,dt$ geometrically from the supplied graph. Part (b) asked whether the length of the line was increasing or decreasing between times $t = 2$ and $t = 3$ hours. Students could determine this by comparing the rate $r(t)$ at which the line grows to 800 people per hour, the rate at which people move from the line onto the ride. Part (c) asked for the time t when the line was longest and the length of the line at that time. Students needed to recognize that the line is growing when $r(t) > 800$ and shrinking when $r(t) < 800,$ so that the line is at its longest during the one time ($t = 3$) when the graph of r decreases through the value $r = 800.$ The number of people waiting in line at that time is computed by subtracting the $3 \cdot 800 = 2400$ people that move from the line onto the ride during the 3 hours from the sum of the 700 people in line at time $t = 0$ and $\int_0^3 r(t)\,dt,$ the number of people joining the line between times $t = 0$ and $t = 3$ hours. Part (d) asked for an equation whose solution gives the earliest time t at which there were no longer people in line. This occurs when the number of people that have joined the line by time $t,$ $700 + \int_0^t r(s)\,ds,$ matches the number that have moved from the line to the ride, $800t.$

Sample: 3A
Score: 9

The student earned all 9 points. In part (c) the student's phrase "so the lineup will be shorter" was ignored.

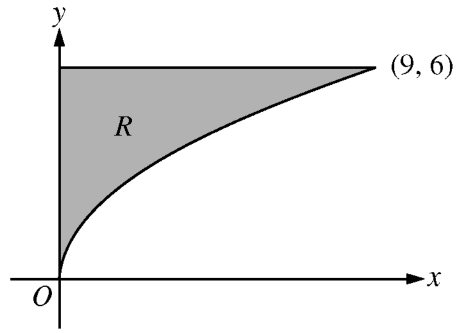
Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student fails to justify "increasing." The student was required to make an explicit comparison of rates, rather than amounts. In part (c) the student earned the first 2 points but does not give a global argument for the justification. In part (d) the student earned the first 2 points.

Sample: 3C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student fails to justify "increasing." In part (c) the student's work is incorrect. In part (d) the student earned the first point for an implicit $800t$ and the second point for the integral. The student's equation does not include the initial condition of 700, so the third point was not earned.

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$.

Volume $= \int_0^6 \frac{3}{16}y^4\, dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

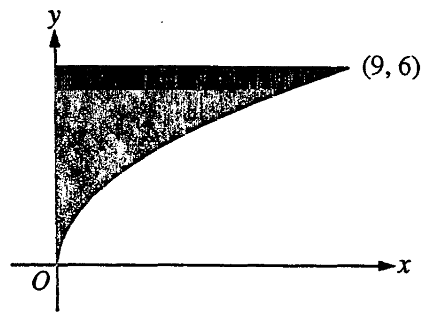
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$R = \int_0^9 6\,dx - \int_0^9 2\sqrt{x}\,dx$

$54 - 2\left[\frac{2}{3}x^{3/2}\Big|_0^9\right]$

$9^{3/2} = 27$

$\frac{27}{3} = 9$

$54 - 2[18 - 0]$

$54 - 36$

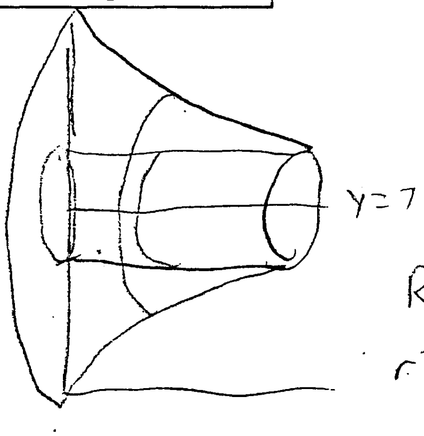
$\boxed{\text{Area} = 18}$ units^2

Do not write beyond this border.

Continue problem 4 on page 1

NO CALCULATOR ALLOWED

Work for problem 4(b)



Vertical Washers $\cancel{\pi(R^2-r^2)}$

dx = thickness

$(\pi R^2 - \pi r^2)\,dx$

$\pi(R^2-r^2)$

$R = 7 - 2\sqrt{x}$

$r = 7-6$

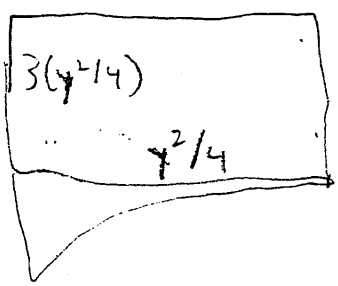
$r = 1$

$\boxed{\text{Volume} = \pi\int_0^9 \left[(7-2\sqrt{x})^2 - 1\right]dx}$

Work for problem 4(c)

$2\sqrt{x} = y \qquad 4\cdot x = y^2$

$x = \frac{y^2}{4}$



$\int_0^6 \left[\frac{y^2}{4}\cdot\frac{3y^2}{4}\right]dy$

Volume of Solid $= \boxed{\int_0^6 \frac{3y^4}{16}\,dy}$

GO ON TO THE NEXT PAGE.

 4B1

NO CALCULATOR ALLOWED

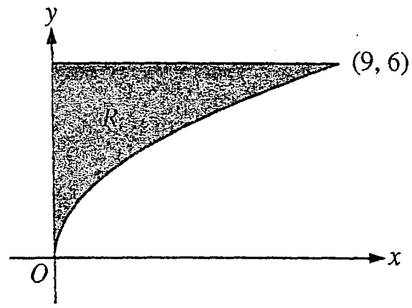
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$R = \int_0^9 (6-2\sqrt{x})^2\,dx = \int_0^9 (36 - 24x^{1/2} + 4x)\,dx$$

$$= \left[36x - 16x^{3/2} + 2x^2\right]_0^9$$

$$= (324 - 432 + 162) - (0)$$

$$= (-108 + 162)$$

$$= \boxed{54 \text{ un.}^2}$$

$36 \times 9 = 324$

$81 \times 9 = 729$

$\sqrt{729} = \sqrt{3\,3\,3\,3\,3\,3} = 27$

$27 \times 16 = 162 + 270 = 432$

Do not write beyond this border.

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$\boxed{V = \pi\int_0^9 \left[(7-2\sqrt{x})^2-(1)^2\right]dx}$$

Work for problem 4(c)

$(y)^2 = (2\sqrt{x})^2$

$y^2 = 4x$

$x = \frac{y^2}{4}$

Height of rectangle $= 3\left(\frac{y^2}{4}\right)$

Base $= \frac{y^2}{4}$

Area of rectangle $= 3\left(\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right)$

$= \frac{3y^4}{16}$

$$\boxed{\text{Volume} = \int_0^6 \left(\frac{3y^4}{16}\right)dy}$$

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Do not write beyond this border.

GO ON TO THE NEXT PAGE.

4C1

NO CALCULATOR ALLOWED

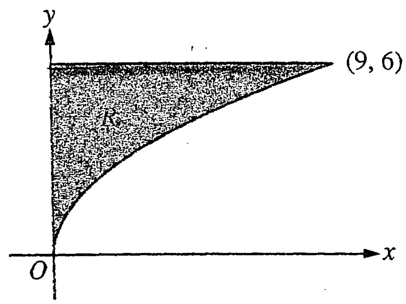
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$\int_0^9 6-2\sqrt{x}\,dx$

$2(x)^{1/2^{-2}}$

$\cancel{2}\cdot\frac{1}{\cancel{2}}x^{-\frac{1}{2}}$

$0-x^{-\frac{1}{2}}\Big|_0^9 = \frac{1}{\sqrt{x}}\Big|_0^9 = \frac{1}{\sqrt{9}} - \frac{1}{\sqrt{0}} = \boxed{\frac{1}{3}}$

$\boxed{R=\frac{1}{3}}$

Do not write beyond this border.

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$\pi\int_0^9 (2\sqrt{x}-7)^2-(6-7)^2\,dx$$

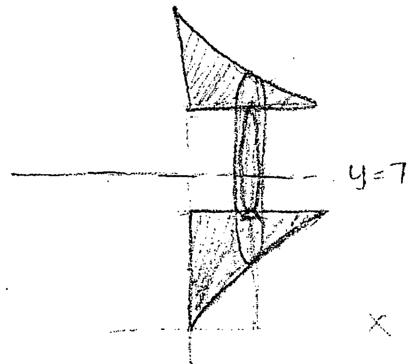
$$\pi\int_0^9 (4x+49-28\sqrt{x})-(1)\,dx$$

$$\pi\int_0^9 4x+48-28\sqrt{x}\,dx$$

$$\pi\left(2x^2+48x-28\cdot\frac{2}{3}(x)^{3/2}\right)\Big|_0^9 \quad (x)^{1/2+1} \quad \frac{3}{2}$$

$$\pi\left(2\cdot 81+48\cdot 9-\frac{56}{3}(9)^{3/2}\right)$$

$$\pi(90)=\boxed{90\pi}$$



$\sqrt{9}\sqrt{9}\sqrt{9}$ 3·3·3

$\sqrt{9^3}$ 9 27

162
422
584
−494
90

56
+4
494

Work for problem 4(c)

$$\pi\int_0^6 3\left((2\sqrt{x})^2-(6)^2\right)dx$$

$$\pi\int_0^6 3(4x-36)\,dx$$

$$\boxed{\pi\int_0^6 (12x-108)\,dx}$$

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Question 4

Overview

In this problem students were given the graph of a region R bounded on the left by the y-axis, below by the curve $y = 2\sqrt{x}$, and above by the line $y = 6$. In part (a) students were asked to find the area of R, requiring an appropriate integral (or difference of integrals), antiderivative and evaluation. Part (b) asked for an integral expression that gives the volume of the solid obtained by revolving R about the line $y = 7$. This is found by integrating cross-sectional areas that correspond to washers with outer radius $7 - 2\sqrt{x}$ and inner radius 1, where $0 \le x \le 9$. Part (c) asked for an integral expression for the volume of a solid whose base is the region R and whose cross sections perpendicular to the y-axis are rectangles of height three times the lengths of their bases in R. Students needed to find the cross-sectional area function in terms of y and use this as the integrand in an integral with lower limit $y = 0$ and upper limit $y = 6$.

Sample: 4A
Score: 9

The student earned all 9 points.

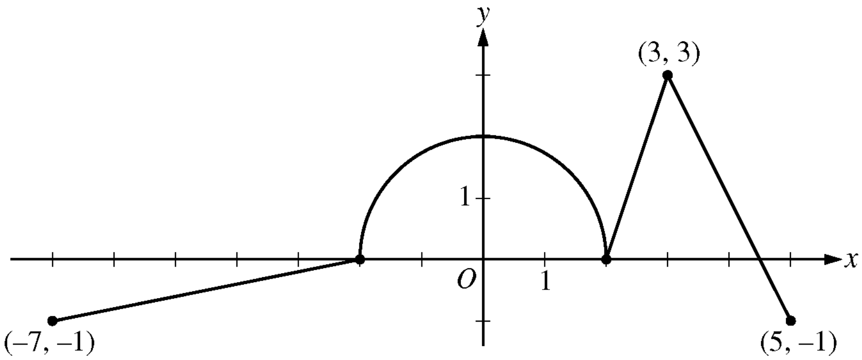
Sample: 4B
Score: 6

The student earned 6 points: no points in part (a), 3 points in part (b), and 3 points in part (c). In part (a) the integrand is shown as the square of the expected integrand, so the student was not eligible for any points. In parts (b) and (c), the student's work is correct.

Sample: 4C
Score: 4

The student earned 4 points: 1 point in part (a), 3 points in part (b), and no points in part (c). In part (a) the student's integrand is correct, but the antiderivative is incorrect; the student differentiated rather than antidifferentiated. No additional points were earned in part (a). In part (b) the student presents an integral in the first line of the solution that earned all 3 points. The student works with the integral, making no errors in lines two and three, and finding an antiderivative in line four. The student's work in lines four and beyond was not evaluated since the question asked for an integral expression only, not for the value of the integral. In part (c) the student's integral was not eligible for any points.

Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

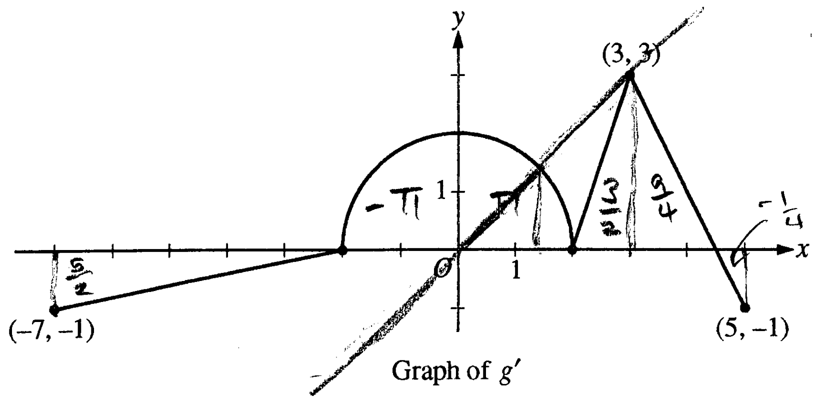
(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$ $g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$	$3: \begin{cases} 1: \text{uses } g(0) = 5 \\ 1: g(3) \\ 1: g(-2) \end{cases}$
(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.	$2: \begin{cases} 1: \text{identifies } x = 0, 2, 3 \\ 1: \text{explanation} \end{cases}$
(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$ On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$. On this interval, $g'(x) = x$ when $x = \sqrt{2}$. The only other solution to $g'(x) = x$ is $x = 3$. $h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$ $h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$ Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.	$4: \begin{cases} 1: h'(x) \\ 1: \text{identifies } x = \sqrt{2}, 3 \\ 1: \text{answer for } \sqrt{2} \text{ with analysis} \\ 1: \text{answer for 3 with analysis} \end{cases}$

NO CALCULATOR ALLOWED



Graph of g'

Work for problem 5(a)

$\boxed{g(0)=5}$

$g(3)-g(0)=\int_0^3 g'(x)\,dx$

$g(3)=5+\int_0^3 g'(x)\,dx$

$g(3)=5+\left(\pi+\frac{3}{2}\right)$

$\boxed{g(3)=\frac{13}{2}+\pi}$

$g(-2)-g(0)=\int_0^{-2} g'(x)\,dx$

$g(-2)=5+\int_0^{-2} g'(x)\,dx$

$g(-2)=5+(-\pi)$

$\boxed{g(-2)=5-\pi}$

Do not write beyond this border.

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Continue problem 5 on page 13.

Work for problem 5(b)

$\boxed{x = 0, 2, 3}$

g'(x) switches from increasing to decreasing or from decreasing to increasing at these three x-values

Work for problem 5(c)

$h(x) = g(x) - \frac{1}{2}x^2$

$h'(x) = g'(x) - x$

Where $h'(x) = 0$...

$g'(x) - x = 0$

$g'(x) = x$

$\boxed{x = \sqrt{2}, 3}$

$x = \sqrt{2}$ is a relative maximum for h(x) because h'(x) switches from positive to negative at this point.

$x = 3$ is not a relative extremum for h(x) because h'(x) does not switch signs.

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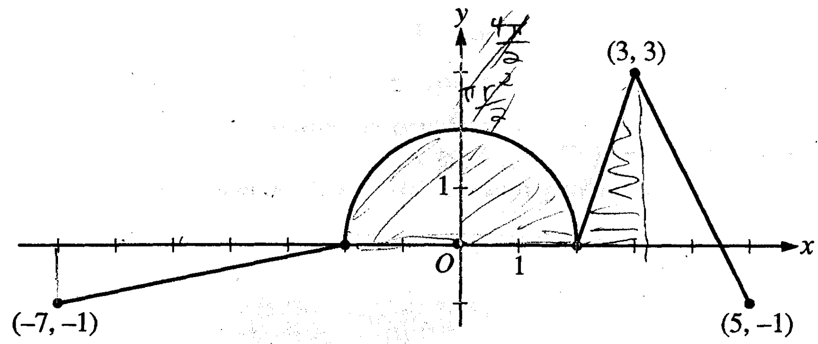
 5B1

NO CALCULATOR ALLOWED



Graph of g'

Work for problem 5(a)

$$g(3) = 5 + \int_0^3 g'(x)\,dx$$

$$= 5 + \left[\left(\tfrac{1}{2}(3)(1)\right) + \left(\tfrac{\pi \cancel{4}}{4}\right)\right]$$

$$= 5 + \frac{3}{2} + \pi$$

$$g(3) = \frac{13}{2} + \pi$$

$$A = \frac{\pi r^2}{4}$$

$$= \frac{2^2 \pi}{4}$$

$$= \pi$$

$$g(-2) = 5 + \int_0^{-2} g'(x)\,dx$$

$$= 5 - \int_{-2}^{0} g'(x)\,dx$$

$$g(-2) = 5 - \pi$$

Do not write beyond this border.

Continue problem 5 on page 13.

Work for problem 5(b)

$g''(x) = 0$ @ $x = 0$

$g''(x) = \text{DNE}$ @ $x = 2$, $x = 3$, $x = -2$

$$\begin{array}{cccccc} & + \nearrow & + \nearrow & - \searrow & + \nearrow & - \searrow \\ -7 & -2 & 0 & 2 & 3 & 5 \end{array}$$

g has a pt of inflection @ $x = 0$, $x = 2$, $x = 3$ b/c g' goes from inc to dec, or vice versa, at those pts.

t -4

Work for problem 5(c)

$g'(x) = x$

$h'(x) = g'(x) - x$

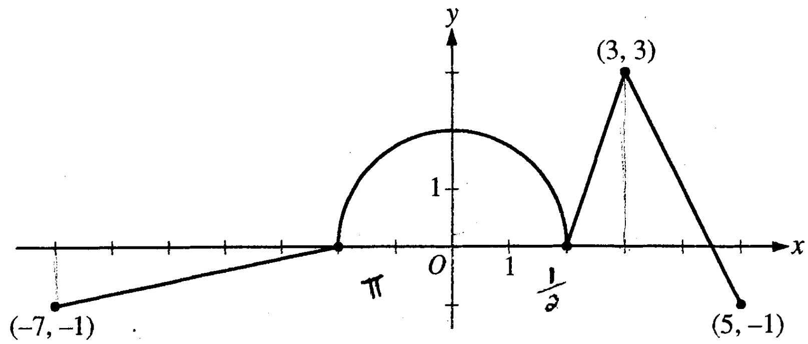
$0 = g'(x) - x$

$h'(x) = 0$ @ $x = 3$

$$\begin{array}{cccc} & + \nearrow & - \searrow & \\ -7 \quad 6 & 3 & 4 \quad 5 \end{array}$$

h has a critical pt @ $x=3$. At $x=3$ h has a rel. max b/c h' goes from pos to neg at that pt.

NO CALCULATOR ALLOWED



Graph of g'

Work for problem 5(a)

$$g(3) = 5 + \int_0^3 g'(x)\,dx$$

$$5 + \left(\tfrac{1}{2}\cdot 3 \cdot 1\right) + \tfrac{1}{4}\pi\, 2^2$$

$$g(3) = 5 + \tfrac{3}{2} + \pi$$

$$g(-2) = 5 - \int_{-2}^{0} g'(x)\,dx$$

$$\pi + \left(7 \cdot 1 \cdot \tfrac{1}{2}\right)$$

$$g(-2) = 5 - \tfrac{7}{2} + \pi$$

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Do not write beyond this border.

Continue problem 5 on page 13.

Work for problem 5(b)

point of Inf. when g'' has a zero or when g' changes sign

g' changes sign at $x=0$, $x=2$ and $x=3$

$\therefore$ g has a point of inf. at $x=0$, $x=2$ and $x=3$

Work for problem 5(c)

critical pt when $h'(t)=0$

$$h'(x) = g'(x) - x$$

critical point at $x=3$

h ↗ ↘

h' + 3 −

$\therefore$ there is a rel. max when $x=3$

GO ON TO THE NEXT PAGE.

Question 5

Overview

This problem described a function g that is defined and differentiable on $[-7, 5]$ and with $g(0) = 5.$ The graph of $y = g'(x)$ on $[-7, 5]$ was given, consisting of three line segments and a semicircle. Part (a) asked for values of $g(3)$ and $g(-2).$ These values are given by $5 + \int_0^3 g'(x)\,dx$ and $5 + \int_0^{-2} g'(x)\,dx,$ respectively, with the definite integrals computed using geometry and properties of definite integrals. Part (b) asked for the x-coordinates of points of inflection for the graph of $y = g(x)$ on the interval $-7 < x < 5.$ Students needed to reason graphically that these occur where the graph of g' changes from increasing to decreasing or vice versa. Part (c) introduced a new function $h(x) = g(x) - \frac{1}{2}x^2$ and asked for x-coordinates of critical points of h and for the classification of each critical point as the location of a relative minimum, relative maximum or neither. Students needed to find that $h'(x) = g'(x) - x$ in order to determine the x-coordinates of critical points and apply a sign analysis of h' to classify these critical points.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (a), 2 points in part (b), and 1 point in part (c). In parts (a) and (b), the student's work is correct. In part (c) the student earned the first point for correctly computing $h'(x).$ Since $x = \sqrt{2}$ is never identified, the student did not earn the second and third points. The fourth point was not earned since the student attempts to justify that $x = 3$ is the x-coordinate of a relative maximum.

Sample: 5C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student earned the first 2 points for correctly using $g(0) = 5$ and computing $g(3).$ The third point was not earned since the value for $g(-2)$ is incorrect. In part (b) the student earned the first point for correctly identifying all three x-coordinates of the points of inflection. The student's statement that "g' changes sign" is not a justification for a point of inflection, so the second point was not earned. In part (c) the student earned the first point for correctly computing $h'(x).$ Since $x = \sqrt{2}$ is never identified, the student did not earn the second and third points. The fourth point was not earned since the student attempts to justify that $x = 3$ is the x-coordinate of a relative maximum.

Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$$\int \frac{1}{y^3}\,dy = \int x\,dx$$

$$-\frac{1}{2y^2} = \frac{x^2}{2} + C$$

$$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$$

$$y^2 = \frac{1}{\frac{5}{4} - x^2}$$

$$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration
Note: $0/5$ if no separation of variables

6A.

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Work for problem 6(a)

at $x=1$ $y=2$

$\frac{dy}{dx} = (1)(2)^3 = 8$ $m=8$

$y = mx + b$

$2 = 8(1) + b$

$b = -6$

at $x=1$
tangent line

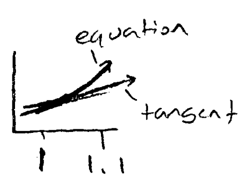
$\boxed{y = 8x - 6}$

Work for problem 6(b)

$y = 8(1.1) - 6$

$y = \boxed{2.8}$

$f(x) > 0$ means all points are positive from $1 < x < 1.1$. The second derivative is also positive from $1 < x < 1.1$ so the concavity is up. ∴ the approximation is an $\boxed{\text{underestimate}}$



Continue problem 6 on page 15

6A₂

Work for problem 6(c)

$\frac{dy}{dx} = xy^3$

$\frac{dy}{y^3} = x\,dx \rightarrow y^{-3}dy = x\,dx$

$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 + C$

$-\frac{1}{2}(2)^{-2} = \frac{1}{2}(1)^2 + C$

$-\frac{1}{8} = \frac{1}{2} + C$

$\boxed{-\frac{5}{8} = C}$

$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 - \frac{5}{8}$

$y^{-2} = -x^2 + \frac{5}{4}$

$\boxed{y = \frac{1}{\sqrt{-x^2 + \frac{5}{4}}}}$

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NO CALCULATOR ALLOWED

6B, 1

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Work for problem 6(a)

a) $x=1$ $\quad \frac{dy}{dx}=(1)(2)^3=8$
$y=2$

$y-2=8(x-1)$

$y=8x-6$

Work for problem 6(b)

b) $y=8(1.1)-6$

$y=8.8-6$

$y=2.8$

$2^3(1+3(1)^2(2)^2)=104$

$+$

It is an over approximation because the f'' at $x=1.1$ is $+$ which means $f(x)$ is concave up.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

6B2

Work for problem 6(c)

c) $\frac{dy}{dx} = xy^3$

$\frac{dy}{y^3} = x\,dx$

$\frac{y^{-2}}{-2} = \frac{x^2}{2} + C$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$\frac{1}{y^2} = -x^2 + C$

$y^2 = \frac{1}{-x^2} + C$

$y = \sqrt{\frac{1}{-x^2}} + C$

$c = 1$

$y = \sqrt{\frac{1}{-x^2}} + 1$

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6C,

Work for problem 6(a)

$\frac{dy}{dx} = xy^3 \quad f(1) = 2$

$1 \; 2^3 \quad \frac{dy}{dx} = 8$

$y - 2 = 8(x - 1)$

Work for problem 6(b)

$y - 2 = 8(1.1 - 1)$

$+2 \quad .8 \quad .1 \quad f(1.1) \approx 2.8$

$y = 2.8$

The approximation would be greater than because this approximation uses the equation of tangent line not the original equation.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

6C2

Work for problem 6(c)

$\frac{dy}{dx} = xy^3$

$\int xy^3\,dx$

$= \frac{1}{2}x^2 \frac{1}{4}y^4$

init cond $f(1)=2 = \frac{1}{2}(1)^2 \cdot \frac{1}{4}(2)^4$

$\frac{1}{2} \cdot \frac{4}{1} = 2$

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Question 6

Overview

This problem identified f as a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2.$ It was also given that solutions to this differential equation satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right).$ Part (a) asked for an equation of the line tangent to the graph of f at $x = 1.$ Students needed to evaluate the given expression for $\frac{dy}{dx}$ at the point $(1, 2)$ to find the slope of this line. Part (b) asked for an approximation to $f(1.1)$ using the tangent line equation from part (a). Given that $f(x) > 0$ for $1 < x < 1.1,$ students were asked to determine whether this approximation is greater than or less than $f(1.1).$ In order to make the determination, students needed to use the given second derivative together with the fact that f is positive on the interval to ascertain the relative position of the tangent line and the graph of $y = f(x)$ for $1 < x < 1.1.$ Part (c) asked for the particular solution $y = f(x)$ with initial condition $f(1) = 2.$ Students should have used the method of separation of variables.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In part (a) the student's work is correct. In part (b) the student earned the approximation point. A local argument for the explanation did not earn the second point. In part (c) the student earned the first 3 points. The student never uses the initial conditions, so no additional points were earned.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a), 1 point in part (b), and no points in part (c). In part (a) the student's work is correct. In part (b) the student earned the approximation point. The student's conclusion is not correct. In part (c) the student never separates the variables and was not eligible for any points.